

1 **An empirically constrained forecasting strategy for**  
2 **induced earthquake magnitudes using extreme value**  
3 **theory**

4 James P. Verdon<sup>1\*</sup>, Leo Eisner<sup>2</sup>

5 *1. School of Earth Sciences, University of Bristol, Wills Memorial Building, Queen's Road,*  
6 *Bristol, United Kingdom.*

7 *2. Seismik s.r.o., Kubišova 1265/8 Praha, Czech Republic.*

8 \* Corresponding Author. Email: James.Verdon@bristol.ac.uk, Tel: 0044 117 331 5135.  
9 Orcid: 0000-0002-8410-2703

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16 **Declaration of Competing Interests**

17 Both authors have acted and continue to act as independent consultants for a variety of organisations  
18 including hydrocarbon operating companies and governmental organisations on issues pertaining to  
19 induced seismicity. None of these organisations had any input into the conception, development,  
20 analysis or conclusions of this study.

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## ABSTRACT

25 *Induced seismicity magnitude models seek to forecast upcoming magnitudes of induced earthquakes during the*  
26 *operation of subsurface industries such as hydraulic fracturing, geothermal stimulation, wastewater disposal,*  
27 *and carbon capture and storage. Accurate forecasting models could guide operational decision-making in real*  
28 *time, for example operations could be reduced or paused if forecast models indicate that magnitudes may exceed*  
29 *acceptable levels. Robust and transparent testing of forecasting models is required if they are to be adopted by*  
30 *operators and regulators of such industries. We develop and test a suite of models based on extreme value*  
31 *estimators to forecast the magnitudes of upcoming induced seismicity events based on observed seismicity. We*  
32 *apply these models to multiple induced seismicity cases from wastewater disposal in Oklahoma and in western*  
33 *Texas, as well as other isolated cases of seismicity caused by subsurface fluid injection in North America,*  
34 *Europe, and China. In total, our testing dataset consists of more than 80 individual sequences of induced*  
35 *seismicity. We find that all the models produce strong correlation between observed and modelled magnitudes,*  
36 *indicating that the forecasting provides useful information about upcoming magnitudes. However, some models*  
37 *are found to systematically over-predict the observed magnitudes, while others tend to under-predict. As such,*  
38 *the combined suite of models can be used to define upper and lower bounds for the expected magnitudes of*  
39 *upcoming events, as well as empirically constrained statistical expectations for how these magnitudes will be*  
40 *distributed between the upper and lower bounds. We conclude by demonstrating how our empirically constrained*  
41 *distribution can be used to produce probabilistic forecasts of upcoming induced earthquake magnitudes,*  
42 *applying this approach to two recent well-known cases of induced seismicity.*

43

## 44 1. INTRODUCTION

45 Cases of induced seismicity have grown rapidly over the past two decades, associated with the growth  
46 and expansion of oilfield technologies such as hydraulic fracturing, wastewater disposal (WWD), and  
47 natural gas storage (NGS). Emerging low-carbon energy technologies such as geothermal and carbon  
48 capture and storage, which entail the injection of fluids into the subsurface, also carry the potential to  
49 generate induced seismicity.

50 In severe cases, induced seismicity has caused damage to nearby buildings and infrastructure, and  
51 injuries to nearby people (e.g., Lee et al., 2019; Lei et al., 2019; Campbell et al., 2020). Even where  
52 induced event magnitudes are insufficient to cause damage, they are nevertheless a source of public  
53 concern (e.g., Evensen et al., 2022). A failure to adequately manage induced seismicity during  
54 development of subsurface geo-energy projects has led to the cancellation of individual projects and  
55 sites, and limits or even moratoria being imposed on entire industries. The need to develop methods to  
56 quantify induced seismicity hazard during operations, primarily by estimating what magnitudes of  
57 earthquakes are likely to be generated, is clear.

58 Our objective is to forecast the expected size of the largest events that will occur during a sequence of  
59 induced seismicity. We refer to the largest event within a given sequence as  $M_{MAX}$ , noting that this  
60 parameter as we use it here is different from the  $M_{MAX}$  parameter used in seismic hazard assessment,  
61 where it denotes the largest magnitude earthquake that could possibly occur given the particular  
62 tectonic circumstances in question (e.g., Mueller, 2010).

63 The magnitude the largest event that occurs during a given industrial operation is of particular concern  
64 to operators and regulators of subsurface industries, since this magnitude will usually determine the  
65 largest ground motion that is generated, and therefore the largest impact to nearby buildings,  
66 infrastructure and people. Accurate forecasting of  $M_{MAX}$  (and preferably, a probability distribution  
67 thereof) could enable operators to make decisions to ensure the safety of their activities by, for  
68 example, reducing, ceasing, or making other mitigation actions to their operations if it becomes likely  
69 that unacceptably high magnitudes will be generated.

70 If induced seismicity forecasting models are to be used to guide decision-making at active industrial  
71 sites, then there is a clear need for robust, transparent testing of such models. Only through robust  
72 testing can we gain confidence in the performance of models such that they can be relied on to guide  
73 operational decisions that, on the one hand, may compromise significant financial investments (if  
74 projects are abandoned due to potential induced seismicity hazard), while on the other hand could  
75 compromise public safety (if larger magnitude events are allowed to occur without mitigation). The  
76 public often takes a strong interest in the occurrence of induced seismicity, and so model testing must  
77 be transparent and reproducible as a loss of trust of public in ability to safely conduct underground  
78 energy operations easily results in loss of social license to operate and rejection of future projects.

79 Empirical testing of forecasting models can go beyond simple assessments of performance since results  
80 can be used to feed back into future forecasts. For example, if a model was observed over a large  
81 number of cases to overpredict the actual  $M_{MAX}$  in say 95 % of cases, then it would be reasonable to  
82 use such a model to define a likely upper bound. Likewise, if a model were observed to underpredict  
83 the actual  $M_{MAX}$  in 95 % of cases, then such a model could be used as a likely lower bound. Rather than  
84 using a single model, a more robust approach is to combine a suite of models, where the respective  
85 performances of each model have been assessed across a large number of cases, in order to produce  
86 an overall forecast for  $M_{MAX}$  that is constrained by empirical observations.

### 87 1.1. Forecasting induced seismicity magnitudes

88 A range of methods to forecast  $M_{MAX}$  during industrial operations has been developed. One approach  
89 is to use numerical geomechanical simulations of subsurface processes (e.g., Rutqvist et al., 2013;

90 Verdon et al., 2015; Dempsey and Suckale, 2017). However, such modelling is often difficult to apply  
91 in practice since a detailed characterisation of the subsurface is required to generate a model. For many  
92 cases of induced seismicity, the causative faults on which seismicity has occurred were not visible in  
93 geophysical surveys acquired prior to the onset of industrial activities (e.g., Eaton et al., 2018; Cesca  
94 et al., 2021; Nantanoi et al., 2022). Even where faults are successfully imaged, quantification of their  
95 mechanical and frictional properties, as required for accurate numerical geomechanical modelling, can  
96 be challenging.

97 The alternative to physics-based numerical modelling is to use statistics-based approaches. For these  
98 methods the observed population of seismic events is characterised statistically, and the statistical  
99 models are then used to make forecasts of the ongoing seismicity. A commonly used approach is to  
100 characterise a relationship between the rate of seismicity and the volume of fluids injected into (or  
101 removed from) the subsurface at an early stage of operations (e.g., McGarr, 1976, Shapiro et al., 2010;  
102 Hallo et al., 2014; Mancini et al., 2021). The future seismicity can then be forecast by extrapolating  
103 this relationship to a future planned injection (or production) volume. This approach has been used to  
104 forecast seismicity and guide decision-making for several notable cases of induced seismicity,  
105 including the Helsinki St1 Deep Heat project (Kwiatek et al., 2019), the Weyburn Carbon Capture and  
106 Storage Project (Verdon, 2016), and during hydraulic fracturing of the Preston New Road shale gas  
107 wells in Lancashire, UK (Clarke et al., 2019; Kettlety et al., 2021). Verdon et al. (2023) published a  
108 comprehensive appraisal of the performance of the Shapiro et al. (2010) and Hallo et al. (2014) models  
109 across a wide range of WWD-induced seismicity case studies.

## 110 **1.2. Forecasting using extreme value estimators**

111 An alternative approach relies solely on the characterisation of the earthquake population, without any  
112 reference to injection or production rates or any other subsurface information. This approach,  
113 developed by Mendecki (2016) for mining induced seismicity, is based on the theory of extreme value  
114 estimators developed by Cooke (1979), and is related to methods developed to estimate tectonic  $M_{MAX}$   
115 values from observed natural earthquake populations (e.g., Kijko, 2004). The relative simplicity of this  
116 method, since it does not require any operational or geological information, makes for an attractive  
117 approach since model performance can be quickly and easily assessed across a broad range (both  
118 geographically and by type of industrial activity) of sites. A limitation of this approach is the need to  
119 for a catalog of observed seismicity to make a forecast. However, we note that for cases of induced  
120 seismicity we often know the complete catalog of induced seismicity to a high magnitude of  
121 completeness if dedicated monitoring systems is usually installed before the start of the activity.

122 There are several ways in which this approach can be applied to forecast  $M_{MAX}$  (see Section 2 for  
123 further details). Mendecki (2016) developed two alternative formulations, the upper limit magnitude,  
124  $M_{UL}$ , which is based on the population of observed magnitudes, and the next record-breaking event,  
125  $M_{JL}$ , which is based on the population of magnitude jumps, with the largest expected magnitude jump  
126 being added to the largest observed event to date. Within these two types of estimates, calculations can  
127 use either the earthquake magnitudes or seismic moments,  $M_O$  (or potencies,  $P = M_O/G$ , where  $G$  is  
128 the shear modulus). Furthermore, the magnitudes and magnitude jumps can be taken from the entire  
129 event catalog sorted into size order, or they can take only the magnitudes and jumps that represent  
130 record-breaking events (i.e., using only the events that represent a new largest event within a  
131 sequence). Given the different ways in which the Cooke (1979) extreme value estimator can be applied  
132 to induced seismicity sequences, there is a clear need to produce a quantitative comparison of their  
133 relative performance in forecasting  $M_{MAX}$  for induced seismicity.

134 Several studies have now applied various versions of the  $M_{UL}$  and/or  $M_{JL}$  method to cases of induced  
135 seismicity (Cao et al., 2020; Verdon and Bommer, 2021; Watkins et al., 2023; Schultz et al., 2023a;  
136 Cao et al., 2023). In general, these studies have produced results that show that, at least from a

137 qualitative perspective, these methods do provide useful forecasting potential, as described in the  
138 following paragraphs.

139 Whereas Mendecki (2016) formulated these methods in terms of seismic potency, all of these later  
140 studies have used earthquake magnitudes. Cao et al. (2020) applied the  $M_{UL}$  and  $M_{JL}$  methods to the  
141 seismicity induced by gas production at Groningen and to a case of hydraulic fracturing-induced  
142 seismicity in North America. In their calculations, they used all events and jumps within the catalogs,  
143 not just record-breaking ones.

144 Verdon and Bommer (2021) applied the  $M_{JL}$  approach to a compilation of 22 instances of hydraulic  
145 fracturing-induced seismicity, and Watkins et al. (2013) applied the  $M_{JL}$  approach to 27 cases of  
146 seismicity induced by WWD and NGS. Like Cao et al. (2020), Verdon and Bommer (2021) and  
147 Watkins et al. (2023) used the jumps between all events (when sorted into size order), not just the  
148 jumps to new record-breaking events.

149 Cao et al. (2023) applied the  $M_{JL}$  approach to 15 cases of induced seismicity (mostly consisting of the  
150 same hydraulic fracturing sequences examined by Verdon and Bommer, 2021), but using as input to  
151 their model only the population of jumps that created new record-breaking events. Schultz et al.  
152 (2023a) applied the  $M_{JL}$  approach to the sequence of WWD-induced seismicity at Musreau Lake,  
153 Alberta. Like Cao et al. (2023), they used as inputs only the population of jumps that created new  
154 record-breaking events.

155 For all the above studies, the assessment of model performance has been somewhat unsystematic.  
156 Mendecki (2016) demonstrated his methods by application to a single example of mining-induced  
157 seismicity but did not make any quantitative assessment of model performance. Likewise, Cao et al.  
158 (2020) and Schultz et al. (2023a) simply compared the evolution of the observed earthquakes with the  
159 changing  $M_{MAX}$  forecasts, noting that the models generally did a reasonable job of fitting the observed  
160 magnitudes. Verdon and Bommer (2021) and Watkins et al. (2023) produced cross-plots of modelled  
161 versus observed magnitudes for the largest event within each sequence, while Cao et al. (2023)  
162 compared modelled and observed magnitudes each time a new largest event occurred. These plots  
163 showed evidence for correlation between observed and modelled magnitudes, but also showed that at  
164 times the  $M_{JL}$  model can underestimate the size of the largest event. As such, there has not yet been  
165 any effort to systematically compare the performance of these methods, either between the different  
166 methods, or for the same method between different sites.

### 167 **1.3. Study objectives**

168 The objective of this study is to provide a systematic assessment of the performance of the Mendecki  
169 (2016)  $M_{UL}$  and  $M_{JL}$  methods as applied to a large number of cases of injection-induced seismicity.  
170 Specifically, we compare the use of earthquake magnitudes versus potencies and we compare the use  
171 of all events and jumps versus the events and jumps that represent new record-breaking events. In  
172 doing so, we investigate influence of these different formulations on the resulting  $M_{MAX}$  forecasts, and  
173 we quantitatively compare their respective performance.

174 Mendecki's (2016) formulations produce a single value for  $M_{MAX}$ . In many cases it may be more  
175 appropriate to produce a probability distribution for the forecast  $M_{MAX}$ . As described above, where  
176 systematic differences in model performance are found, observations across a large number of sites  
177 could be used to define an empirically constrained probability distribution for  $M_{MAX}$ . For instance, if  
178 one approach was found to systematically underestimate  $M_{MAX}$ , while another method was found to  
179 systematically overestimate  $M_{MAX}$ , then these two values could be used to estimate upper and lower  
180 bounds for the expected  $M_{MAX}$ . Having produced quantitative assessments of model performance in the

181 first part of our paper, we go on to investigate whether the different approaches to forecasting  $M_{MAX}$   
 182 can be combined to produce an empirically constrained probabilistic forecasting approach.

## 183 2. METHODS

184 Mendecki (2016) described two approaches to forecasting induced seismicity magnitudes using the  
 185 order statistics theory of Cooke (1979). For a random sample of  $n$  magnitude (or potency) observations,  
 186  $M^O$ , drawn from a constant underlying distribution, the upper limit for future such observations can be  
 187 estimated as:

$$188 \quad M_{UL} = 2M_n^O - \sum_{i=1}^{n-1} \left[ \left(1 - \frac{i}{n}\right)^n - \left(1 - \frac{i+1}{n}\right)^n \right] M_{n-i}^O, \quad (1)$$

189 where  $M_i^O$  represents the event magnitudes (or potencies) sorted into size order, from smallest to  
 190 largest, such that  $M_n^O$  is the largest event observed to date, which we refer to as  $M_{MAX}^O$ .

191 Alternatively, one can consider the jumps in magnitude (or potency) between events,  $\Delta M^O$ , since an  
 192 estimate for the next largest event can be obtained by adding the estimated maximum jump,  $\Delta M_{MAX}$ ,  
 193 to the observed largest event. We refer to this estimate as the “jump-limited” maximum magnitude:

$$194 \quad M_{JL} = M_{MAX}^O + \Delta M_{MAX}. \quad (2)$$

195 The estimated maximum magnitude (or potency) jump is calculated using the same formulation as  
 196 Equation 1, but applied to the distribution of magnitude jumps:

$$197 \quad \Delta M_{MAX} = 2\Delta M_{n_j}^O - \sum_{i=1}^{n_j-1} \left[ \left(1 - \frac{i}{n_j}\right)^{n_j} - \left(1 - \frac{i+1}{n_j}\right)^{n_j} \right] \Delta M_{n_j-i}^O, \quad (3)$$

198 where  $\Delta M_i^O$  represents the magnitude (or potency) jumps ordered from smallest to largest, and  $n_j$  is  
 199 the number of jumps.

### 200 2.1. Modelling approaches

201 Equations 1 – 3 describe two approaches to estimating  $M_{MAX}$ , which we refer to hereafter with the  
 202 subscripts  $M_{UL}$  and  $M_{JL}$ , respectively. These calculations can be applied to observed magnitude or  
 203 potency values. Hereafter, we refer to results computed using magnitudes with the subscript  $_{MM}$ , and  
 204 results computed using potencies with the subscript  $_{MO}$ .

205 As described in Section 1.2, these methods have been applied using the full earthquake catalogs, where  
 206  $M_i^O$  represents the entire event population sorted into size order and  $\Delta M_i^O$  represents the magnitude  
 207 (or potency) jump between every event when the entire population is sorted into magnitude order, with  
 208  $\Delta M_i^O$  then being sorted into size order. Alternatively, one can use an event population that consists  
 209 only of the record-breaking events as they appear in the sequence, where  $M_i^O$  represents the record-  
 210 breaking events sorted into size order, and  $\Delta M_i^O$  represents the jumps between the record-breaking  
 211 events. Hereafter, we refer to calculations using the entire event population resorted into size order  
 212 with the subscript  $_{AE}$  (for all events) and calculations using only the record-breaking events as  $_{RB}$  (for  
 213 record-breaking events).

214 We note that dedicated microseismic monitoring arrays often produce large numbers of events (e.g.,  
 215 Verdon and Budge, 2018), but even for a very large catalog ranging across several orders of magnitude  
 216 we typically observe only a few record-breaking events. Thus, the methods based on record-breaking  
 217 versus all events represent different approaches to statistical estimates. By definition, the record-  
 218 breaking method excludes aftershocks as these are smaller than, and occur after, a mainshock and

219 therefore do not contribute to record-breaking series. However, the approach based on all events  
 220 includes aftershocks in the evaluation of the maximum magnitude while representing whole sequence.  
 221 These combinations mean that we have a total of 8 possible ways in which  $M_{MAX}$  can be estimated.  
 222 These are summarised in Table 1. In the following section we introduce the datasets that we use to  
 223 assess the performance of each method, before presenting our results in Section 4.

224

225 *Table 1: Summary of different model implementations used for  $M_{MAX}$  forecasting*

Model No.	Model Name	Upper Limit [UL] or Jump-Limited [JL] formula	All Events in Size Order [AE] or Record Breaking only [RB]	Magnitudes [MM] or Potencies [MO]
1	$M_{UL\_RB\_MM}$	UL	RB	MM
2	$M_{UL\_RB\_MO}$	UL	RB	MO
3	$M_{UL\_AE\_MM}$	UL	AE	MM
4	$M_{UL\_AE\_MO}$	UL	AE	MO
5	$M_{JL\_RB\_MM}$	JL	RB	MM
6	$M_{JL\_RB\_MO}$	JL	RB	MO
7	$M_{JL\_AE\_MM}$	JL	AE	MM
8	$M_{JL\_AE\_MO}$	JL	AE	MO

226

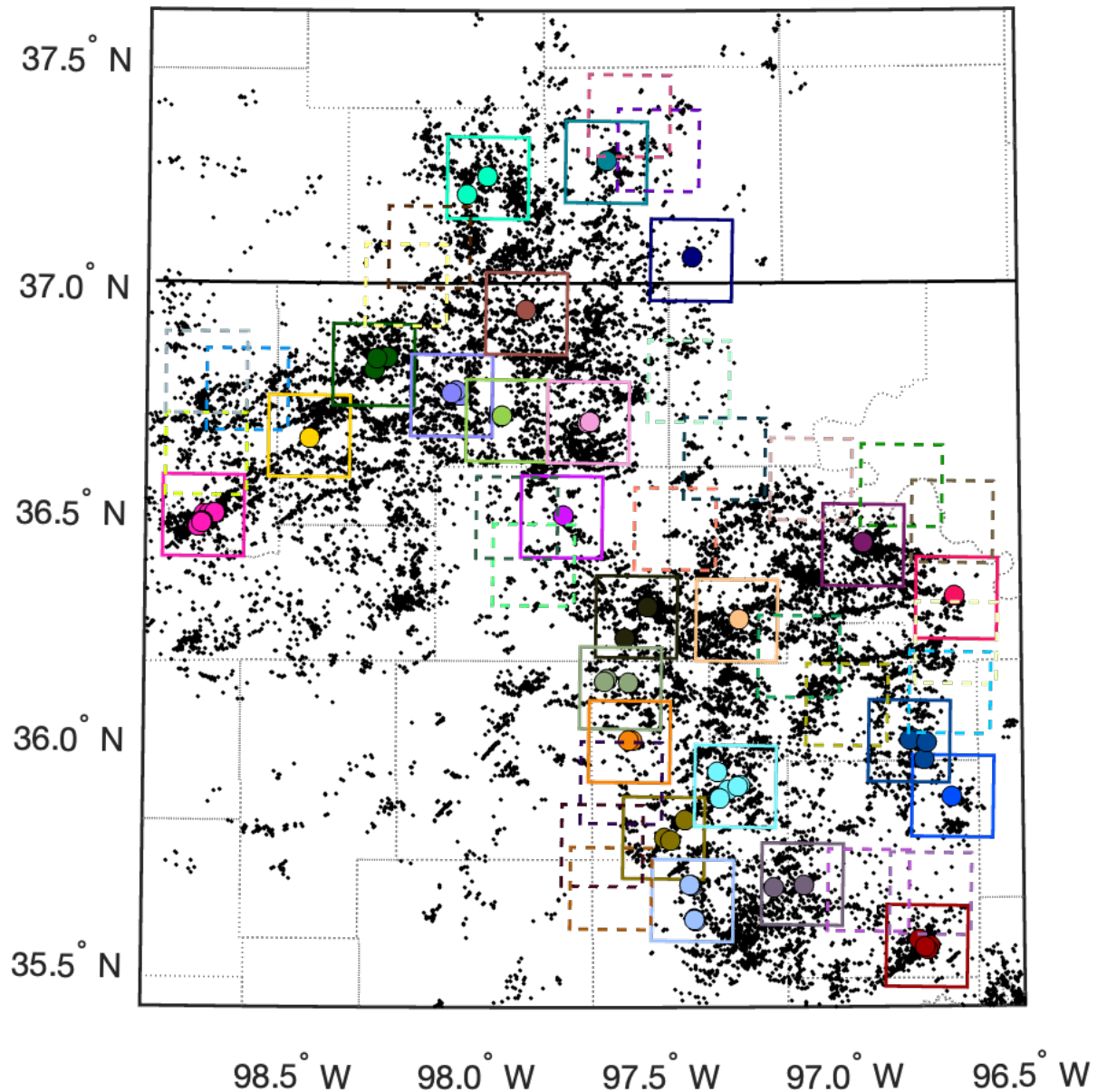
## 227 3. DATASETS

### 228 3.1. Oklahoma and southern Kansas

229 WWD in central and northern Oklahoma and southern Kansas (OK-KS hereafter) has increased  
 230 significantly over the past two decades, driven primarily by a move towards hydrocarbon production  
 231 from reservoirs with high water fractions, with the produced water then requiring disposal (Rubenstein  
 232 and Mahani 2015). WWD has caused significant amounts of induced seismicity (Weingarten et al.  
 233 2015), including some of the largest induced events to have ever been recorded from fluid injection  
 234 activities, such as the M 5.6 Prague (Keranen et al., 2013) and M 5.8 Pawnee (Yeck et al., 2017)  
 235 sequences. Induced seismicity in Oklahoma has also been caused by hydraulic fracturing (e.g.,  
 236 Holland, 2013; Skoumal et al., 2018; Verdon and Rodríguez-Pradilla, 2023), particularly in the  
 237 Anadarko Basin. However, our focus here is on central and northern Oklahoma and southern Kansas,  
 238 where the bulk of the seismicity is caused by WWD.

239 In this study we use the earthquake catalog published by Park et al. (2022), who used the PhaseNet  
 240 deep learning model (Zhu and Beroza, 2019) to detect earthquakes recorded by publicly available  
 241 seismic networks in the OK-KS region. The deep learning model produced a significant increase in  
 242 event detection, improving detection thresholds by at least 1 magnitude unit over pre-existing  
 243 earthquake catalogs for the region. We adopt a minimum magnitude of completeness of  $M_C = 1.5$ ,  
 244 based on the magnitude-frequency relationships plotted in Figure 2 of Park et al. (2022). To estimate  
 245 potencies from the given magnitudes, we adopt a single value of  $G = 20$  GPa (this value is adopted for  
 246 all sequences in our study).

247 There are 70 earthquakes in the Park et al. (2022) catalog with magnitudes  $\geq 4.0$ . Some of these events  
 248 occur in close proximity to each other such that they can be considered to be part of the same sequence.  
 249 Through examination of the spatial and temporal evolution of the seismicity in OK-KS, we identified  
 250 24 individual sequences in which induced event magnitudes reached or exceeded  $M 4.0$  (see Figure  
 251 1). We take these 24 sequences as test datasets for our analysis. For each case, we define a  $20 \times 20$  km  
 252 square around the  $M \geq 4.0$  event (or the largest event for sequences which contain more than one  
 253  $M \geq 4.0$  event). All earthquakes within this square are taken as representing part of the sequence and  
 254 used to perform our  $M_{MAX}$  forecasting. The  $M \geq 4.0$  events, and the  $20 \times 20$  km squares around them,  
 255 are shown in Figure 1.



256

257 *Figure 1: Map of the OK-KS study area. Black dots show all earthquakes with  $M \geq 1.5$  and coloured*  
 258 *circles show events with  $M \geq 4.0$ . The solid boxes show the  $20 \times 20$  km blocks around each of the*  
 259 *sequences containing  $M \geq 4.0$  events, while the dashed boxes show  $20 \times 20$  km blocks in which 500*  
 260 *events were recorded with no  $M \geq 3.5$  events. The box colours used in this figure correspond to the*  
 261 *marker colours used in Figure 3.*

262

263 In testing induced seismicity forecasting models, there can be a tendency to focus on cases where  
 264 larger magnitude events occurred, since these cases tend to attract the most attention (from the public  
 265 and policy makers, as well as from academics). However, comprehensive testing should include  
 266 sequences that did not reach larger magnitudes, since our objective is to develop models that can  
 267 differentiate between sequences that do, and that do not, escalate to higher magnitude events. Hence,  
 268 in addition to the 24 sequences with  $M \geq 4.0$  events, we identify the same number of cases where  
 269 magnitudes did not exceed  $M 3.5$ , selecting twenty-four  $20 \times 20$  km blocks at random within the study  
 270 area that contained at least 500 events but no events with  $M \geq 3.5$ . To do so, we randomly generated



271 block positions and rejected those that did not meet these criteria, continuing until we had 24 cases.  
272 The 24 blocks without larger magnitude events are also shown in Figure 1.

### 273 **3.2. Permian Basin, western Texas**

274 Induced seismicity has been recognised in the Permian Basin of western Texas (WTX hereafter) since  
275 the 1970s (Davis and Pennington, 1989). Rates of seismicity in the basin have increased substantially  
276 since 2015 (Skoumal et al., 2020), associated with WWD and hydraulic fracturing. Given the co-  
277 location of these activities, distinguishing causality between WWD and hydraulic fracturing can be  
278 challenging, although the bulk of the seismicity is thought to have been caused by WWD (Grigoratos  
279 et al., 2022). Three  $M \geq 5.0$  events have been induced in this basin: the March 2020  $M$  5.0 event near  
280 to the city of Pecos in Reeves County (Skoumal et al., 2021), the November 2022 Coalson Draw  $M$  5.4  
281 event in western Reeves County, and the December 2022  $M$  5.2 event in Martin County, just to the  
282 north of the city of Midland (Hennings and Young, 2023).

283 In this study we use the TexNet earthquake catalog (Savvaidis et al., 2019), with data running from  
284 the start of 2017 until April 2023. We adopt a minimum magnitude of completeness of  $M_C = 2.0$  from  
285 inspection of the magnitude-frequency relationship for this catalog. There are 48 events for which  
286  $M \geq 4.0$  (Figure 2). Our examination of the temporal and spatial evolution of the seismicity identified  
287 11 individual sequences in which induced event magnitudes reached or exceeded  $M$  4.0. Much like for  
288 our OK-KS datasets, we define  $20 \times 20$  km squares around each sequence and use all events within  
289 these blocks to perform our  $M_{MAX}$  forecasting. We then identify an equal number (i.e., 11) of  $20 \times 20$   
290 km blocks containing at least 100 events (we use a lower criterion here recognising the lower number  
291 of events in the TexNet catalog compared to the Park et al. (2022) catalog for OK-KS) but no events  
292 larger than  $M$  3.5, in order to test  $M_{MAX}$  model performance for cases where larger magnitude events  
293 did not occur.

### 294 **3.3. Watkins et al. (2023) sequences**

295 Watkins et al. (2023) published  $M_{MAX}$  forecasts using the  $M_{JL\_SO\_MM}$  formulation for more than 20  
296 individual sequences of WWD and NGS-induced seismicity. Some of the Watkins et al. (2023)  
297 sequences are already included in our OK-KS and WTX datasets described in the previous sections  
298 (Reeves and Cogdell in Texas, Cushing, Fairview, Guthrie-Langston, Pawnee and Prague in  
299 Oklahoma, Milan and Harper in Kansas), while for some older sequences with lower levels of  
300 monitoring, the largest events occurred before a sufficient number of events were available to compute  
301  $M_{MAX}$  estimates (e.g., the Cordel sequence in Alberta). This left 16 additional sequences which we were  
302 able to include in our analysis, including: the Azle-Reno, Dallas-Fort Worth, Venus, Timpson and  
303 Irving sequences in eastern Texas (Hennings et al., 2021; Frohlich et al., 2014); the Guy-Greenbrier  
304 sequence in Arkansas (Horton, 2012); the Youngstown sequence in Ohio (Kim, 2013); the Paradox  
305 Valley, Greeley and Raton Basin sequences in Colorado (Block et al., 2014; Yeck et al., 2016; Nakai  
306 et al., 2017); the Eagle West, Graham, and Musreau Lakes sequences in western Canada (Horner et  
307 al., 1994; Hosseini and Eaton, 2018; Li et al., 2022); the Rongchang sequence in the Sichuan Basin  
308 (Wang et al., 2020); the Castor project in the Gulf of Valencia, Spain (Cesca et al., 2021); and the  
309 Puerto Gaitán sequence, Colombia (Molina et al., 2020). For each of these sequences, we use the  
310 earthquake catalogs published in the Supplementary Materials of Watkins et al. (2023). We refer to  
311 these sequences as the W23 cases hereafter.

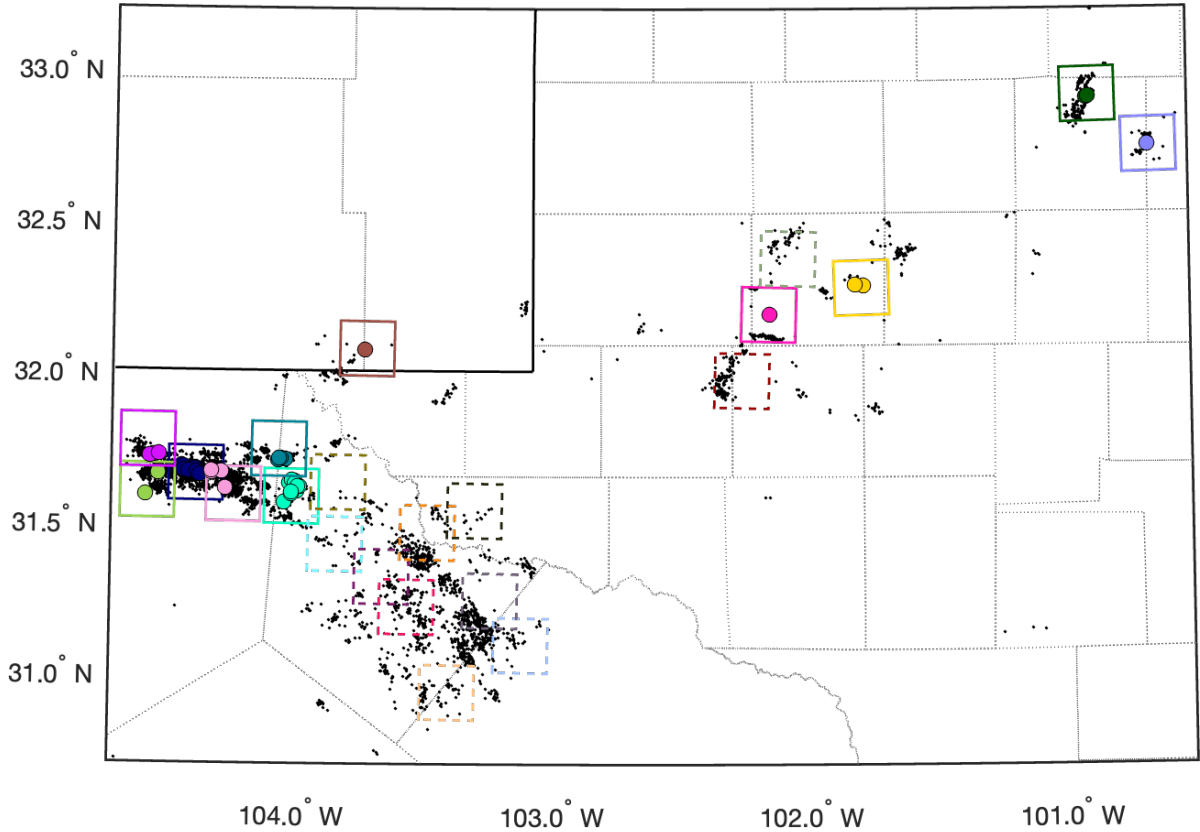
### 312 **3.4. Application**

313 For the OK-KS and WTX datasets we compute  $M_{MAX}$  values at intervals of 0.5 months, starting at the  
314 time when at least 10 events above the magnitude of completeness within the sequence have been  
315 recorded. For the W23 sequences, the timespans of each sequence are highly variable – we therefore

316 compute  $M_{MAX}$  values at 1,000 evenly-spaced intervals between the first and final event within each  
 317 sequence. For each time interval we compute modelled maximum magnitude values,  $M_{MAX}^M$ , using all  
 318 the events in the sequence that occurred prior to the given time.

319 Our objective is to assess the forecast performance as each sequence evolves. We therefore make  
 320 comparisons between observed and modelled magnitudes each time there is a new largest event within  
 321 the sequence. Each new largest event within the sequence is treated as an observed maximum  
 322 magnitude event,  $M_{MAX}^O$ . The  $M_{MAX}^O$  values are compared against the  $M_{MAX}^M$  values calculated at the  
 323 timestep prior to when the  $M_{MAX}^O$  event occurred.

324



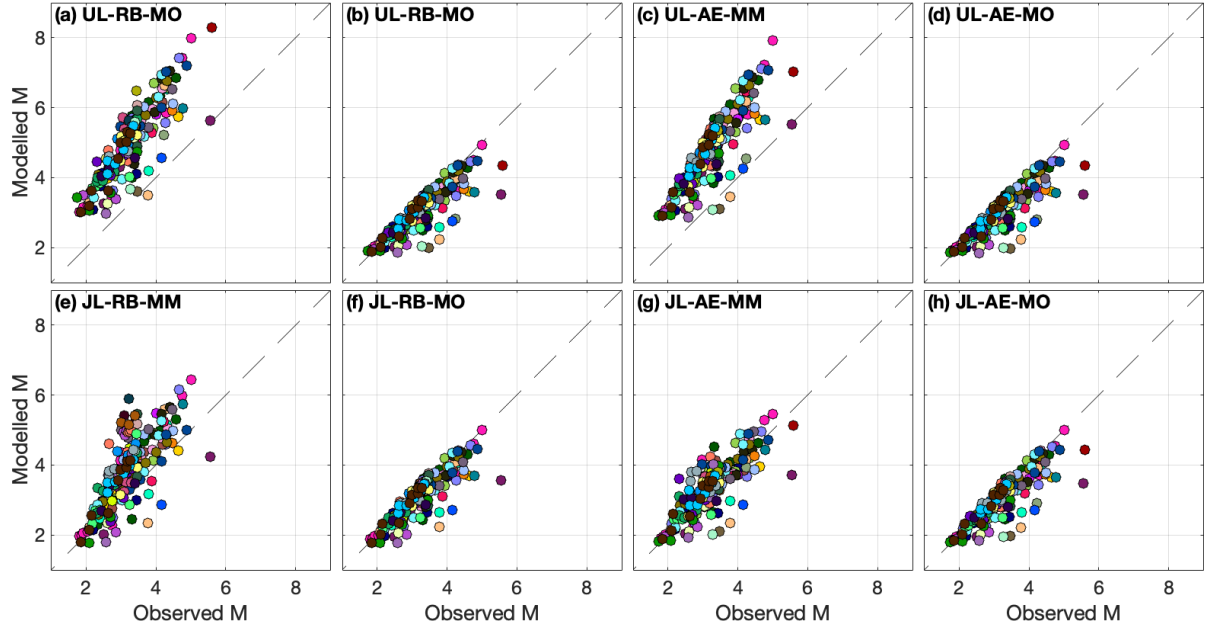
325

326 *Figure 2: Map of the western Texas study area. Black dots show all earthquakes with  $M \geq 1.0$  and*  
 327 *coloured circles show events with  $M \geq 4.0$ . The solid boxes show the  $20 \times 20$  km blocks around each*  
 328 *of the sequences containing  $M \geq 4.0$  events, while the dashed boxes show  $20 \times 20$  km blocks in which*  
 329 *100 events were recorded with no  $M \geq 3.5$  events. The box colours used in this figure correspond to*  
 330 *the marker colours used in Figure 4.*

331

## 332 4. RESULTS

333 Figures 3, 4 and 5 show our results, comparing the observed and forecast  $M_{MAX}^O$  and  $M_{MAX}^M$  values,  
 334 using each of the 8 methods described in Table 1, for the sequences from OK-KS (Figure 3), WTX  
 335 (Figure 4), and the W23 sequences (Figure 5). In total we have applied our models to 86 total sequences  
 336 (48 in OK-KS, 22 in WTX, 16 from W23), with a combined total of 331 individual record-breaking  
 337 events within these sequences (205 from OK-KS, 72 from WTX, 54 from W23).



338

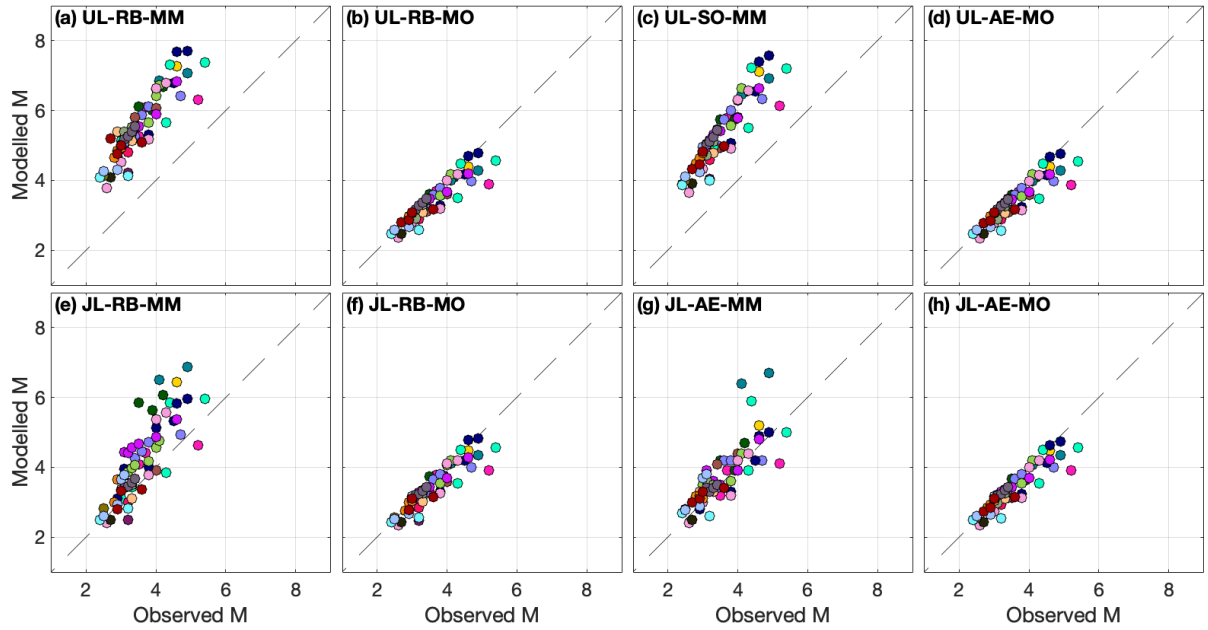
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341

342

Figure 3: Results for OK-KS sequences comparing observed and modelled magnitudes for each of the  $M_{MAX}$  forecasting methods listed in Table 1. Marker colours correspond to sequences within each box shown in Figure 1.



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Figure 4: Results for WTX sequences comparing observed and modelled magnitudes for each of the  $M_{MAX}$  forecasting methods listed in Table 1. Marker colours correspond to sequences within each box shown in Figure 2.

348

349

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351

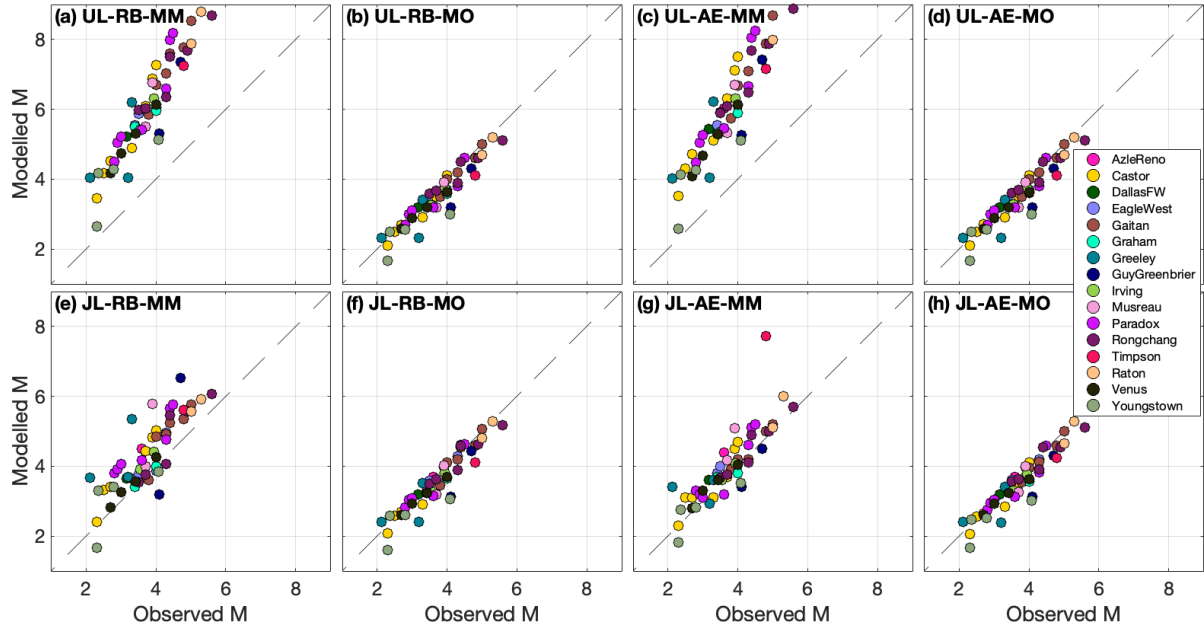
352

353

We quantify the model performance using several metrics. We compute the root-mean-squared (RMS) error between modelled and observed magnitudes,  $\sigma_{RMS}$ , the Pearson correlation coefficient between modelled and observed magnitudes,  $r$ , and the line of (least squares) best-fit,  $m$ . A well-performing model should minimise  $\sigma_{RMS}$  and maximise  $r$ , and have a best-fit line close to 1.0, implying a 1:1 relationship between  $M_{MAX}^M$  and  $M_{MAX}^O$ . Additionally, in most applications we anticipate that  $M_{MAX}$  forecasting will be used to guide operational decision making in order to avoid unwanted large events.

354 It is therefore of particular importance that models do not make large underpredictions, such that the  
 355 actual seismicity significantly exceeds what has been forecast by the model. We therefore compute  
 356  $N_{UP}$ , the percentage of  $M_{MAX}^O$  instances where the forecast  $M_{MAX}^M$  value was a significant  
 357 underprediction with  $M_{MAX}^M < M_{MAX}^O - 0.5$ . These metrics are listed in Table 2 for the OK-KS, WTX  
 358 and the W23 sequences respectively.

359



360

361 *Figure 5: Results for the W23 sequences comparing observed and modelled magnitudes for each of*  
 362 *the  $M_{MAX}$  forecasting methods listed in Table 1.*

363

364 In general, we observe strong correlation between the modelled and observed  $M_{MAX}$  values, implying  
 365 that these methods all provide useful forecasting information for induced seismicity magnitudes, and  
 366 could therefore be used as part of a decision-making strategy to manage induced seismicity. The  
 367 performance of these models is generally better than that found by Verdon et al. (2023) for commonly  
 368 used volume-based forecasting models, having higher correlation coefficients between modelled and  
 369 observed magnitudes, lower RMS errors (except for the  $M_{UL\_RB\_MM}$  and  $M_{UL\_RB\_MM}$  models, see below),  
 370 and fewer cases where models produced significant underpredictions of upcoming magnitudes.

371 More detailed inspection of Figures 3 – 5 and Table 2 leads us to the following conclusions, all of  
 372 which are consistent between the OK-KS, WTX, and W23 sequences.

373 **Using re-sorted magnitudes or just record-breaking events does not significantly change**  
 374 **forecasting performance.** The use of the entire earthquake catalog, versus solely using record-  
 375 breaking events (or jumps to record-breaking events), was a key point of difference between Cao et al.  
 376 (2020), Verdon and Bommer (2021) and Watkins et al. (2023) on the one hand, and Cao et al. (2023)  
 377 and Schultz et al. (2023a) on the other. However, comparison of panels (a) vs (c), (b) vs (d), (e) vs (g),  
 378 and (f) vs (h) of Figures 3 – 5 show that these different implementations in fact produce very similar  
 379 results. Examination of Equations 1 and 3 show that this outcome is unsurprising, since only the first  
 380 few terms of the weighting applied to the summation of the magnitudes (or jumps), given by:

381 
$$W = \left(1 - \frac{i}{n}\right)^n - \left(1 - \frac{i+1}{n}\right)^n, \quad (4)$$

382 are significant (Mendecki, 2016). The first weightings correspond to the largest magnitudes (or  
 383 magnitude jumps), which tend to be magnitudes (or jumps) that produce record-breaking events.  
 384 Figure 6 plots the value of  $W$  as a function of  $i$ , in this case setting  $n = 20$ . The weighting term drops  
 385 to values of 0.01 or less after the 4<sup>th</sup> term in the summation (the weighting applied to the 4<sup>th</sup>-largest  
 386 magnitude or jump).

387

388

Table 2: Performance metrics for OK-KS, WTX and W23 sequences.

Model	$\sigma_{RMS}$	$r$	$m$	$N_{UP}$ [%]
<b>OK-KS</b>				
$M_{UL\_RB\_MM}$	1.84	0.86	1.27	0
$M_{UL\_RB\_MO}$	0.41	0.86	0.76	14.2
$M_{UL\_AE\_MM}$	1.67	0.86	1.24	0
$M_{UL\_AE\_MO}$	0.41	0.85	0.76	14.2
$M_{JL\_RB\_MM}$	0.93	0.75	1.11	3.4
$M_{JL\_RB\_MO}$	0.37	0.87	0.82	12.7
$M_{JL\_AE\_MM}$	0.47	0.81	0.85	7.3
$M_{JL\_AE\_MO}$	0.41	0.85	0.78	14.6
<b>WTX</b>				
$M_{UL\_RB\_MM}$	2.06	0.90	1.23	0
$M_{UL\_RB\_MO}$	0.32	0.92	0.78	12.5
$M_{UL\_AE\_MM}$	1.84	0.91	1.26	0
$M_{UL\_AE\_MO}$	0.32	0.92	0.78	12.5
$M_{JL\_RB\_MM}$	0.89	0.83	1.35	2.8
$M_{JL\_RB\_MO}$	0.32	0.91	0.81	12.5
$M_{JL\_AE\_MM}$	0.54	0.80	0.98	5.6
$M_{JL\_AE\_MO}$	0.32	0.91	0.79	12.5
<b>W23</b>				
$M_{UL\_RB\_MM}$	2.37	0.93	1.62	0
$M_{UL\_RB\_MO}$	0.34	0.94	0.93	11.1
$M_{UL\_AE\_MM}$	2.43	0.92	1.66	0
$M_{UL\_AE\_MO}$	0.34	0.94	0.93	11.1
$M_{JL\_RB\_MM}$	0.81	0.83	1.04	3.7
$M_{JL\_RB\_MO}$	0.34	0.93	0.94	11.1
$M_{JL\_AE\_MM}$	0.59	0.85	1.05	3.7
$M_{JL\_AE\_MO}$	0.34	0.94	0.93	9.3

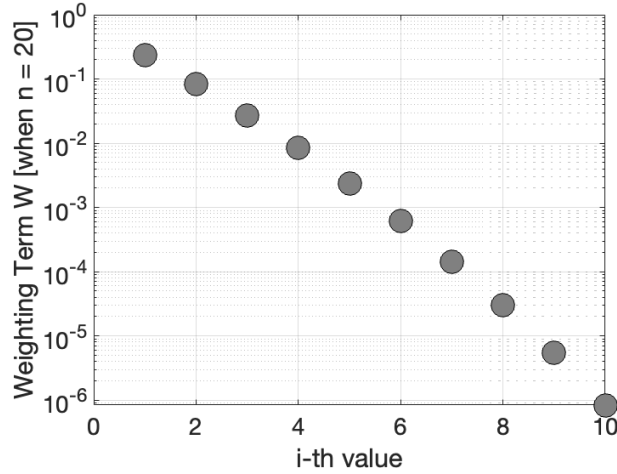
389

390 **Upper limit models using magnitude provide a credible upper limit.** The  $M_{UL\_AE\_MM}$  and  $M_{UL\_RB\_MM}$   
 391 models (panels (a) and (c) in Figures 3 – 5) did not produce any significant underpredictions ( $N_{UP} = 0$ ).  
 392 This is notable given that we have applied it to 86 individual earthquake sequences. Hence, the  
 393  $UL\_MM$  values (upper limit calculations using magnitudes) do seem to provide a credible upper limit  
 394 to induced earthquake magnitudes.

395 However, while these values never produced underpredictions, they did not provide a good fit to  
 396 observations overall, tending to produce significant overpredictions in most cases. As a result, the  
 397  $M_{UL\_AE\_MM}$  and  $M_{UL\_RB\_MM}$  models gave the largest  $\sigma_{RMS}$  values, and best-fit relationships with  $m$   
 398 significantly higher than 1.0. That said, the correlation coefficients for the  $M_{UL\_AE\_MM}$  and  $M_{UL\_RB\_MM}$

399 models are not significantly worse than those of other models, implying that the scatter between  
 400 modelled and observed magnitudes is no worse than for the other models, just the fit is not along the  
 401 1:1 line, resulting in systematic overprediction.

402



403

404 *Figure 6: Value of the weighting  $W$  applied within the summation term in Equations 1 and 3 (as*  
 405 *defined in Equation 4) as a function of  $i$ , where  $n$  is set at 20.*

406

407 **Next record-breaking models using magnitudes produce the highest scatter.** Although the  
 408  $M_{JL\_AE\_MM}$  and  $M_{JL\_AE\_MM}$  models (panels (e) and (g) in Figures 3 – 5) produced reasonable fits between  
 409 observed and modelled magnitudes, with  $m$  values close to 1.0, these models had the lowest correlation  
 410 coefficients of all the models, and the highest  $\sigma_{RMS}$  values with the exception of the overpredicting  
 411  $M_{UL\_AE\_MM}$  and  $M_{UL\_RB\_MM}$  models, as described above. The  $M_{JL\_AE\_MM}$  and  $M_{JL\_RB\_MM}$  models therefore  
 412 produced the highest scatter between modelled and observed magnitudes and may therefore have the  
 413 least utility in forecasting. This is ironic given that this approach has been the most widely used to  
 414 date, forming the basis of results presented by Cao et al. (2020; 2023), Verdon and Bommer (2021),  
 415 Watkins et al. (2023) and Schultz et al. (2023a).

416 **Potency-based models have the least scatter, but significantly underpredict on occasion.** All four  
 417 of the models that used earthquake potencies,  $M_{UL\_AE\_MO}$ ,  $M_{UL\_RB\_MO}$ ,  $M_{JL\_AE\_MO}$ , and  $M_{JL\_RB\_MO}$  (panels  
 418 (b), (d), (f) and (h) in Figures 3 – 5) produced very similar results. These models had the lowest  $\sigma_{RMS}$   
 419 values and highest correlation coefficients, indicating that these models had low scatter and the closest  
 420 match between modelled and observed magnitudes. However, these models also produced the largest  
 421 number of underpredictions, with between 10 – 15 % of events being underpredicted by more than 0.5  
 422 magnitude units. We surmise that in most cases where sequences are evolving relatively gently, the  
 423 potency-based models perform well. However, they do not perform as well in capturing the more  
 424 unusual sequences where a sharp increase in magnitudes takes place.

## 425 5. DISCUSSION

### 426 5.1. Towards an empirically constrained probabilistic model

427 Our results show that the upper limit magnitude-based models,  $M_{UL\_AE\_MM}$  and  $M_{UL\_RB\_MM}$ , provided  
 428 credible upper bounds for the actual event magnitudes, having no significant underpredictions after  
 429 application to a large number of sequences. However, in most cases these models overpredicted the  
 430 observed events. In contrast, the potency-based models ( $M_{UL\_AE\_MO}$ ,  $M_{UL\_RB\_MO}$ ,  $M_{JL\_AE\_MO}$ , and

431  $M_{JL\_RB\_MO}$ ) generally produced a good fit to the observed magnitudes, but occasionally produced  
 432 significant underpredictions.

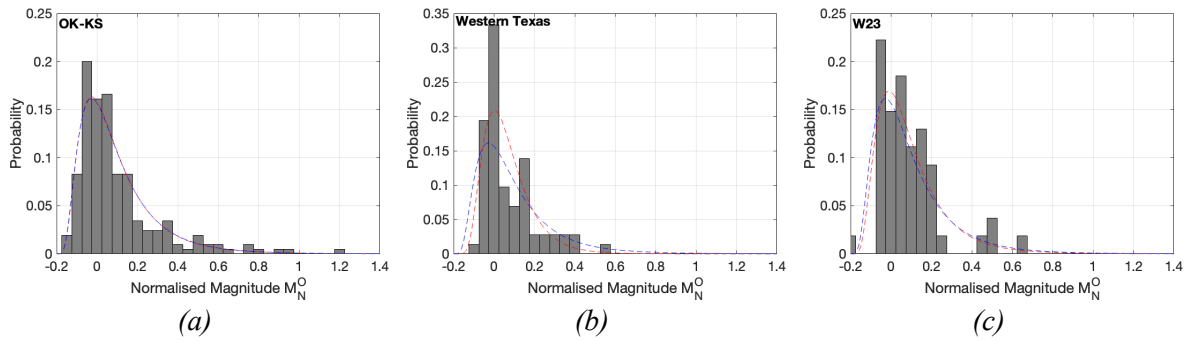
433 From this, it is reasonable to propose a composite approach to forecasting event magnitudes where  
 434  $M_{UL\_AE\_MM}$  or  $M_{UL\_RB\_MM}$  is used provide an upper bound to the expected magnitude,  $M_{UB}$ , and  
 435  $M_{UL\_AE\_MO}$ ,  $M_{UL\_RB\_MO}$ ,  $M_{JL\_AE\_MO}$ , or  $M_{JL\_RB\_MO}$  is used to provide a lower bound for the expected  
 436 magnitude,  $M_{LB}$ . Hereafter, we use  $M_{UL\_RB\_MM}$  for the upper bound, and  $M_{JL\_AE\_MO}$  for the lower bound.

437 The probability distribution of event magnitudes between these bounds can be estimated through  
 438 empirical calibration with our observed seismicity. For each event, we normalise the observed event  
 439 magnitude relative to the  $M_{LB}$  and  $M_{UB}$  values at the time of the event's occurrence:

$$440 \quad M_N^O = \frac{M_{MAX}^O - M_{LB}}{M_{UB} - M_{LB}} \quad (5)$$

441 We then examine the distribution of these normalised magnitudes – where do events typically fall with  
 442 respect to the upper and lower magnitude bounds? Our results for each of our studies are shown in  
 443 Figure 7.

444



445 *Figure 7: Distribution of normalised observed magnitudes  $M_N^O$  (bars) where the observed*  
 446 *magnitudes are normalised relative to the modelled upper and lower bound estimates, for the OK-KS*  
 447 *(a), WTX (b), and W23 (c) sequences. The red dashed lines show the best-fit shifted lognormal*  
 448 *distribution for each case, while the blue dashed lines show a shifted lognormal distribution with*  
 449  *$\mu_{LN} = -1.4$  and  $\sigma_{LN} = 0.6$ .*

450

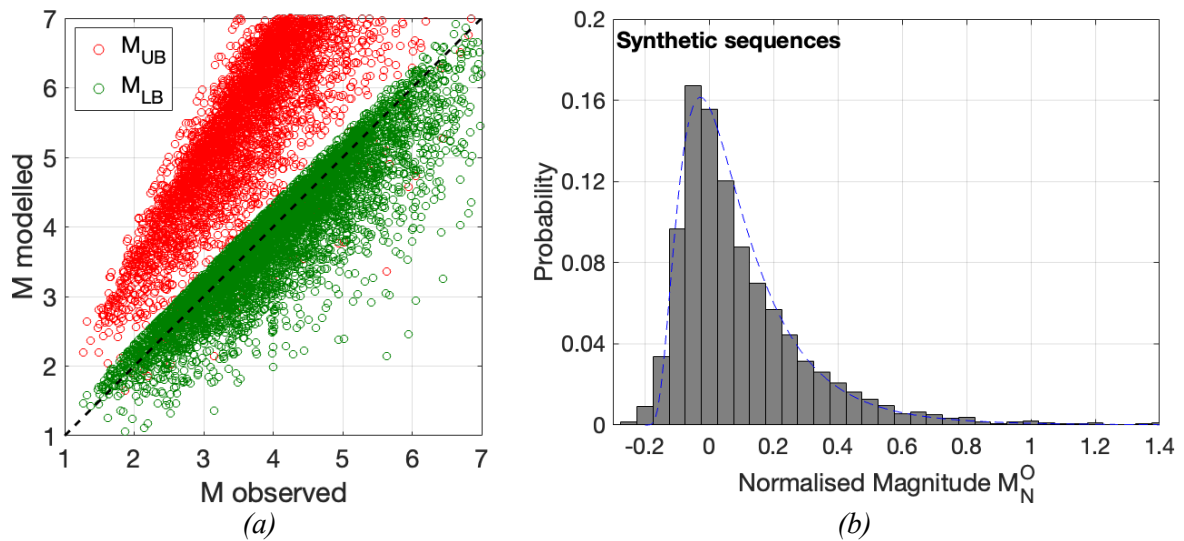
451 The distributions of  $M_N^O$  are remarkably consistent between the three sets of sequences that we studied.  
 452 Most values are close to 0, i.e., they match the modelled lower bound values,  $M_{MAX}^O = M_{LB}$ . However,  
 453 the distribution has a tail of higher values extending towards 1, i.e., where observed magnitudes reach  
 454 towards the higher bound values,  $M_{MAX}^O = M_{UB}$ .

455 We find that the observed  $M_N^O$  distributions are well modelled by a shifted lognormal distribution. The  
 456 red curves in Figure 7 show the best-fit lognormal curves for each set of sequences after applying a  
 457 shift of 0.2 units to  $M_N^O$ . The best fit lognormal means ( $\mu_{LN}$ ) and deviations ( $\sigma_{LN}$ ) for the OK-KS, WTX  
 458 and W23 sequences are, respectively,  $\mu_{LN} = [-1.4, -1.41, -1.37]$  and  $\sigma_{LN} = [0.59, 0.43, 0.54]$ .

459 The similarities of these values suggest a fundamental underlying property is controlling the  
 460 distribution of observed magnitudes relative to the modelled upper and lower bounds. We further  
 461 investigate this with some synthetic testing. We generate 1,000 earthquake sequences from an  
 462 underlying Gutenberg-Richter relationship with a  $b$  value of 1.0. For each synthetic sequence, the total  
 463 number of events is drawn at random from a uniform distribution between 500 – 10,000, and the  
 464 minimum magnitude is drawn random from a uniform distribution between 0.5 – 2.5. These  
 465 distributions reflect the range of event numbers and magnitudes of completeness from our compilation  
 466 of observed induced seismicity sequences. The timing of each event in the sequence is drawn at random

467 from a range from 0 – 1. Having created synthetic earthquake sequences, we then apply the  $M_{UB}$  and  
 468  $M_{LB}$  calculations as done for our real cases, computing these values at the time before each new largest  
 469 event within the sequence is observed.

470 Our results are shown in Figure 8. The distribution of  $M_{UB}$  and  $M_{LB}$  values relative to the ‘observed’  
 471 (i.e., simulated) magnitudes is consistent with the observed cases presented in Figures 3 – 5, with the  
 472  $M_{UB}$  values generally larger than the observed magnitudes and very rarely producing underestimates  
 473 while the  $M_{LB}$  values are generally close to the observed values, but occasionally produce significant  
 474 underestimations. Figure 8b shows the distribution of  $M_N^O$  values for our synthetic data, which again  
 475 look very similar to our observed sequences, being well described by a shifted lognormal distribution  
 476 with a shift of -0.2 units, and  $\mu_{LN} = -1.4$  and  $\sigma_{LN} = 0.6$ . This modelled distribution is shown by the blue  
 477 dashed curve in Figure 8b, and is also reproduced as a blue dashed curve in Figures 7(a-c) to facilitate  
 478 comparison with the observed  $M_N^O$  distributions.



479  
 480 *Figure 8: Results for synthetic seismicity sequences. In (a) we compare the ‘observed’ (i.e.,*  
 481 *simulated) magnitude values with the modelled  $M_{UB}$  (red) and  $M_{LB}$  (green) values. In (b) we plot the*  
 482 *distribution of normalised magnitude values ( $M_N^O$ ) (bars), and the best-fit shifted lognormal*  
 483 *distribution (blue dashed line).*

484  
 485 As can be seen in Figure 7, the shifted lognormal distribution produced by our synthetic modelling  
 486 provides a good match to the observed values. These results, and in particular the consistency found  
 487 for  $M_N^O$  between our different case studies and our synthetic models, enables us to construct an  
 488 empirically constrained probabilistic model for induced seismicity forecasting using extreme value  
 489 estimators. For a given sequence of seismicity, we compute the  $M_{UB}$  and  $M_{LB}$  bounds at a given time.  
 490 Note that, given the shifted lognormal distribution for  $M_N^O$  that we have adopted,  $M_N^O$  can go below a  
 491 value of 0, and above a value of 1, so  $M_{UB}$  and  $M_{LB}$  are not truly bounds since there are low but non-  
 492 zero probabilities that  $M_{MAX}^O < M_{LB}$  or  $M_{MAX}^O > M_{UB}$ . However, we refer to them as bounds nonetheless  
 493 since the majority of events will fall between these values.

494 Having computed  $M_{UB}$  and  $M_{LB}$ , we can compute the probability for the next largest magnitude event  
 495 that will occur in the sequence. For a given magnitude, we use Equation 5 to normalise that magnitude  
 496 relative to  $M_{UB}$  and  $M_{LB}$ , and then estimate the probability of occurrence for that event from a shifted  
 497 lognormal distribution, with a shift of -0.2 units,  $\mu_{LN} = -1.4$ , and  $\sigma_{LN} = 0.6$ .



498 **5.2. Application to out-of-sample cases**

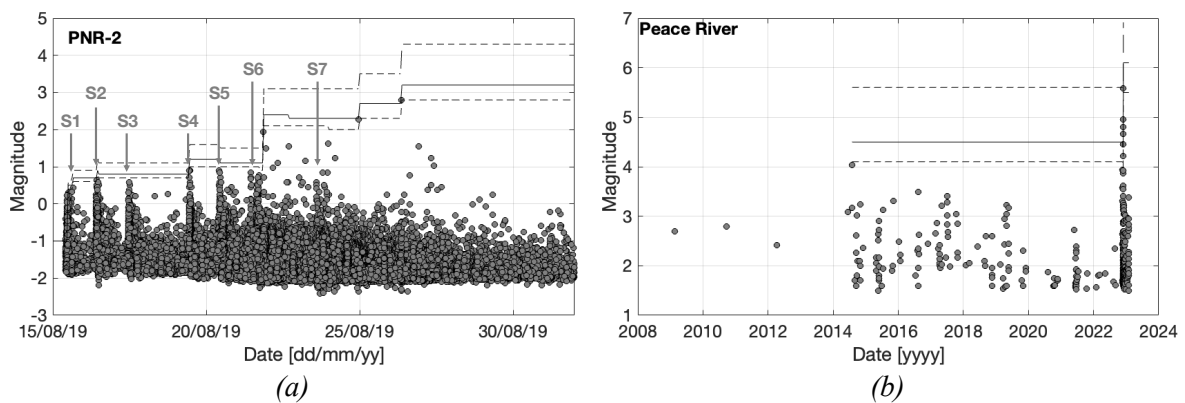
499 We demonstrate this approach by application to two notable cases of induced seismicity: from  
500 hydraulic fracturing at the Preston New Road PNR-2 well in Lancashire, England in 2019 (Kettley et  
501 al., 2021), and from seismicity associated with WWD activities in northwestern Alberta, near to the  
502 town of Peace River (Schultz et al., 2023b). The PNR-2 sequence is notable because its occurrence led  
503 the UK government to impose a moratorium on hydraulic fracturing, primarily because of the  
504 perceived inability to “accurately predict the probability or magnitude of earthquakes linked to  
505 fracking operations” (BEIS, 2019).

506 The Peace River sequence reached a magnitude of  $M$  5.6 in November 2022, making it the largest  
507 magnitude induced event in the Western Canada Sedimentary Basin. This sequence is useful for our  
508 purposes since, given when it occurred, it was not included in the sequences compiled by Watkins et  
509 al. (2023), and so it represents an out-of-sample test, since the sequences in W23 were used to generate  
510 our empirically constrained distribution of  $M^O_N$ .

511 For the PNR-2 sequence, we use the corrected moment magnitudes published by Kettley and Butcher  
512 (2022) – note that these  $M_W$  values are different from the  $M_L$  values published by Kettley et al. (2021).  
513 For the Peace River sequence, we use earthquakes from the Alberta Geological Survey database (AGS,  
514 2020).

515 Our results are shown in Figure 9, where the observed seismicity is compared with the forecast values.  
516 The solid lines in Figure 9 show the magnitude with a 50 % chance of exceedance,  $M_{50}$ , while the  
517 dashed lines show  $M_{95}$  and  $M_{05}$  (i.e., the magnitude that has a 95 % chance of being exceeded, and the  
518 magnitude that has a 5 % chance of being exceeded).

519



520 *Figure 9: Application of the empirically constrained forecasting model to the Preston New Road*  
521 *PNR-2 (a) and Peace River (b) sequences. Observed events are marked with grey dots. The solid line*  
522 *marks  $M_{50}$ , while the dashed lines mark  $M_{05}$  and  $M_{95}$ . For PNR-2, the bursts of seismicity associated*  
523 *with each discrete hydraulic fracturing interval (Stages 1-7) are marked with grey arrows.*

524

525 For PNR-2, the  $M$  2.8 event is well within the forecast range, and close to the  $M_{50}$  value at the time it  
526 occurred. Hydraulic fracturing at PNR-2 was conducted as a series of discrete injection stages,  
527 typically lasting between 1 – 2 hours, with only one injection stage taking place each day. Stage 7 was  
528 the last stage to have been stimulated, with the  $M$  2.8 event occurring roughly 72 hours after this stage  
529 had been completed (Kettley et al., 2021). The forecast values prior to Stage 7 are therefore of  
530 particular interest since these values could have informed the operational decision to perform this  
531 stage. At the time that injection of Stage 7 began, the likelihood of reaching or exceeding  $M$  2.8 was  
532 12 %. The forecasting model therefore provides a reasonable characterisation of the hazard at the time  
533 that the decision to proceed with Stage 7 was made.

534 Interestingly, the event that most exceeds the forecast is the  $M$  1.9 event that followed Stage 6. At the  
535 start of injection of Stage 6, the likelihood of reaching or exceeding  $M$  1.9 was only 1 %. Kettlety et  
536 al. (2021) identified that Stage 6 saw a significant change in geomechanical behaviour in the reservoir,  
537 with microseismicity beginning to occur along the fault structure that ultimately hosted the  $M$  2.8  
538 event. Kettlety et al. (2021) interpreted the microseismicity prior to Stage 6 as being associated with  
539 hydraulic fracture propagation (and the reactivation of some natural fracture networks), whereas  
540 microseismicity from Stage 6 onwards begins to represent the onset of reactivation of a critically  
541 stressed fault. This highlights one of the challenges with induced seismicity forecasting – where a  
542 sudden change in the underlying geomechanical behaviour takes place, events from prior to this change  
543 may not be useful in forecasting subsequent behaviour. For PNR-2, the microseismicity associated  
544 with hydraulic fracture propagation during the earlier stages does not do a good job of forecasting what  
545 happens as the fault begins to reactivate. Once the fault begins to reactivate the forecasting model  
546 using the seismicity from this point onwards does a good job of forecasting the subsequent seismicity  
547 that then develops. This observation shows that care should be taken to fully interpret and understand  
548 the geomechanical behaviours that can be manifested in microseismic event observations when using  
549 catalogs to forecast induced seismicity hazard.

550 For the Peace River case, the forecast values are stable for the duration of the sequence. The  $M$  5.6  
551 event that occurs is close to the  $M_{05}$  value, indicating a 5 % likelihood of this magnitude being reached  
552 or exceeded.

### 553 **5.3. Time dependent forecasting**

554 The forecasting methods developed here do not provide any estimate of whether a new record-breaking  
555 event will occur and, if so, when it will occur. The timing of the next record-breaking event could be  
556 estimated from the growing number of earthquakes within a sequence. Mendecki (2016) shows that  
557 the expected number of record-breaking events,  $N_{rb}$ , in a population of  $n$  events can be approximated  
558 as:

$$559 \quad N_{rb} \approx \ln(n) + 0.577215, \quad (6)$$

560 with the variance given by:

$$561 \quad \text{Var}(N_{rb}) = \ln(n) - 1.0677. \quad (7)$$

562 The number of record-breaking events relative to the total number of events within the sequence could  
563 therefore be used to indicate whether another record-breaking event might be imminent. Further  
564 investigation of this possibility is clearly merited.

565 Perhaps more importantly, the methods developed here, which are based on the concept of record-  
566 breaking events, imply that  $M_{MAX}$  for a sequence of induced seismicity will be ever-increasing. In  
567 practice, many sequences of induced seismicity generated by long-term injection have shown time-  
568 dependent behaviour where magnitudes increased during the first years of injection, but then stabilised  
569 and decreased over time (Watkins et al., 2023; Verdon et al., 2023).

570 As sequences stabilise and abate, magnitude forecasts based on extreme value estimators will cease to  
571 be appropriate. Clearly, some means of estimating the point at which the rates and magnitudes of  
572 induced seismicity are no longer increasing is required. One method may be to compare the numbers  
573 of record-breaking events when the sequence is run forwards versus when the sequence is run in a  
574 time-reversed order (Mendecki, 2016). If the earthquake sequence is sampling from an underlying  
575 stationary distribution, then we would expect the same number of record-breaking events whether the  
576 sequence is run forwards or backwards. If there are significantly more record-breaking events when  
577 the sequence is run forwards, then this would imply that the hazard is increasing, while if there are

578 significantly more record-breaking events when the sequence is run in reverse, then this would imply  
579 that the hazard is abating. Again, further investigation of this concept is clearly merited.

## 580 **6. CONCLUSIONS**

581 We have assessed the performance of induced seismicity forecasting models for  $M_{MAX}$  using methods  
582 based on extreme value estimators. These models can be implemented in a number of different ways,  
583 and we have quantitatively compared the performance of these implementations. We compiled a  
584 database of over 80 individual sequences of induced seismicity against which comparisons of model  
585 performance were made. We found that using all events within a catalog or just the record-breaking  
586 events made little difference to the forecasting results, since the models are primarily sensitive to the  
587 largest magnitude events in the sequence.

588 Estimates of  $M_{MAX}$  using the upper limit method with event magnitudes tended to overestimate the  
589 observed magnitudes. However, unlike other models, this model never significantly underpredicted  
590 the observed seismicity, so it has use in defining an upper bound for  $M_{MAX}$ . The models which used  
591 earthquake potency instead of magnitude produced the closest overall fit to the observed magnitudes,  
592 but on occasion did produce significant underestimates of the observed magnitudes. The potency-  
593 based models seldom produced overpredictions of the observed magnitudes.

594 Based on these observations, we conclude that the upper limit magnitude-based model and the jump-  
595 limited potency-based models can be combined to give upper and lower bounds for the upcoming  
596 events within an induced seismicity sequence. We found that most of the observed events were much  
597 closer to the lower bound magnitude estimates. We used this observation to define an empirically  
598 constrained probability distribution for expected magnitudes relative to the estimated upper and lower  
599 bounds. This distribution was consistent between the different populations of induced seismicity  
600 sequences compiled for our analysis, as well as for sequences that were generated synthetically.

601 We applied this forecasting approach to two out-of-training-sample (i.e., not used in defining our  
602 empirically constrained distribution) sequences of induced seismicity. We find that in both cases our  
603 modelling approach does a good job of characterising the induced seismicity that occurred. However,  
604 the example from PNR-2 again highlights one of the major challenges in forecasting induced  
605 seismicity: where rapid changes in the underlying geomechanical processes occur (such as when a  
606 different fault begins to be perturbed), seismicity from earlier within the sequence may not be useful  
607 for forecasting once this change has occurred.

608

## 609 **Data and Resources**

610 The earthquake catalog for Oklahoma was sourced from Park et al. (2022), where the catalog is  
611 provided as a digital supplement. The earthquake catalog for Texas was sourced from the TexNet  
612 database at <https://www.beg.utexas.edu/texnet-cisr/texnet/earthquake-catalog> (last accessed  
613 14/02/2024). The earthquake catalogs for the sequences described by Watkins et al. (2023) are  
614 available as a digital supplement to that paper. The catalog for PNR-2 is available from the UK  
615 National Geoscience Data Centre at  
616 <https://webapps.bgs.ac.uk/services/ngdc/accessions/index.html#item173104> (last accessed  
617 14/02/2024). The catalog for the Peace River sequence was sourced from the Alberta Earthquake  
618 Dashboard at [https://ags-aer.shinyapps.io/Seismicity\\_waveform\\_app/](https://ags-aer.shinyapps.io/Seismicity_waveform_app/) (last accessed 14/02/2024).

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